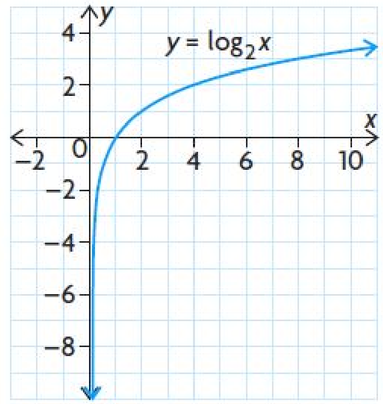
Chapter 7 Problem Set – Exponential and Logarithmic Functions

7.1 Exploring the Logarithmic Function #1c, 4, Finish 5 – 7, 9, 11 (pg 451 in textbook)

1. Sketch a graph of the inverse of each exponential function.

c) $f(x) = \left(\frac{1}{3}\right)^x$



4. Explain how you can use the graph of $y = \log_2 x$ (at right) to help you determine the solution to $2^y = 8$.

5. Write the equation of the inverse of each exponential function in exponential form.

a) $y = 3^x$ c) $y = \left(\frac{1}{4}\right)^x$

b) $y = 10^x$ d) $y = m^x$

6. Write the equation of the inverse of each exponential function in question 5 in logarithmic form.

7. Write the equation of each of the following logarithmic functions in exponential form.

a) $y = \log_5 x$ c) $y = \log_3 x$

b) $y = \log_{10} x$ d) $y = \log_{\frac{1}{4}} x$

9. Evaluate each of the following:

a) $\log_2 4$ c) $\log_4 64$ e) $\log_2\left(\frac{1}{2}\right)$

b) $\log_3 27$ d) $\log_5 1$ f) $\log_3 \sqrt{3}$

11. For each of the following logarithmic functions, write the coordinates of the five points that have y-values of -2, -1, 0, 1, 2.

a) $y = \log_2 x$ b) $y = \log_{10} x$

7.2 Evaluating Logarithms (Part 1) #3 (pg 466 in textbook)

3. Evaluate.

a) $\log_5 5$ c) $\log_2\left(\frac{1}{4}\right)$ e) $\log_{\frac{2}{3}}\left(\frac{8}{27}\right)$

b) $\log_7 1$ d) $\log_7 \sqrt{7}$ f) $\log_2 \sqrt[3]{2}$

7.3 Evaluating Logarithms (Part 2) #4 – 7, 9, 10, 11, 14 – 17, 19 (pg 466 in textbook)

4. Solve for x. Round your answers to two decimal places, if necessary.

a) $\log\left(\frac{1}{10}\right) = x$ c) $\log(1\ 000\ 000) = x$ e) $\log x = 0.25$

b) $\log 1 = x$ d) $\log 25 = x$ f) $\log x = -2$

5. Evaluate.

a) $\log_6 \sqrt{6}$ c) $\log_3 81 + \log_4 64$ e) $\log_5 \sqrt[3]{5}$

b) $\log_5 125 - \log_5 25$ d) $\log_2 \frac{1}{4} - \log_3 1$ f) $\log_3 \sqrt{27}$

6. **K** Use your knowledge of logarithms to solve each of the following equations for x.

a) $\log_5 x = 3$ c) $\log_4 \frac{1}{64} = x$ e) $\log_5 x = \frac{1}{2}$

b) $\log_x 27 = 3$ d) $\log_{\frac{1}{4}} x = -2$ f) $\log_4 x = 1.5$

7. Graph $f(x) = 3^x$. Use your graph to estimate each of the following logarithms.

a) $\log_3 17$ b) $\log_3 36$ c) $\log_3 112$ d) $\log_3 143$

9. Evaluate.

a) $\log_3 3^5$ c) $4^{\log_4 \frac{1}{16}}$ e) $a^{\log_a b}$

b) $5^{\log_5 25}$ d) $\log_m m^n$ f) $\log_{\frac{1}{10}} 1$

10. Evaluate $\log_2 16^{\frac{1}{3}}$.

11. **A** The number of mold spores in a petri dish increases by a factor of 10 every week. If there are initially 40 spores in the dish, how long will it take for there to be 2000 spores?

14. The function $S(d) = 93 \log d + 65$ relates the speed of the wind, S, in miles per hour, near the centre of a tornado to the distance that the tornado travels, d, in miles.

a) If a tornado travels a distance of about 50 miles, estimate its wind speed near its centre.

b) If a tornado has sustained winds of approximately 250 mph, estimate the distance it can travel.

15. The astronomer Johannes Kepler (1571–1630) determined that the time, D, in days, for a planet to revolve around the Sun is related to the planet's average distance from the Sun, k, in millions of kilometres. This relation is defined by the equation $\log D = \frac{3}{2} \log k - 0.7$. Verify that Kepler's equation gives a good approximation of the time it takes for Earth to revolve around the Sun, if Earth is about 150 000 000 km from the Sun.

16. Use Kepler's equation from question 15 to estimate the period of revolution of each of the following planets about the Sun, given its distance from the Sun.

a) Uranus, 2854 million kilometres

b) Neptune, 4473 million kilometres

17. **T** The doubling function $y = y_0 2^{\frac{t}{D}}$ can be used to model exponential growth when the doubling time is D. The bacterium *Escherichia coli* has a doubling period of 0.32 h. A culture of *E. coli* starts with 100 bacteria.

a) Determine the equation for the number of bacteria, y, in x hours.

b) Graph your equation.

c) Graph the inverse.

d) Determine the equation of the inverse. What does this equation represent?

e) How many hours will it take for there to be 450 bacteria in the culture? Explain your strategy.

19. **C** Consider the expression $\log_5 a$.

a) For what values of a will this expression yield positive numbers?

b) For what values of a will this expression yield negative numbers?

c) For what values of a will this expression be undefined?

7.4 The Laws of Logarithms #2, 4 – 8, 10, 11 (pg 475 in textbook)

2. Express each of the following as a logarithm of a product or quotient.

a) $\log 5 + \log 7$

b) $\log_3 4 - \log_3 2$

c) $\log_m a + \log_m b$

d) $\log x - \log y$

e) $\log_6 7 + \log_6 8 + \log_6 9$

f) $\log_4 10 + \log_4 12 - \log_4 20$

4. **K** Use the laws of logarithms to simplify and then evaluate each expression.

a) $\log_3 135 - \log_3 5$

b) $\log_5 10 + \log_5 2.5$

c) $\log 50 + \log 2$

d) $\log_4 4^7$

e) $\log_2 224 - \log_2 7$

f) $\log \sqrt{10}$

5. Describe how the graphs of $y = \log_2(4x)$, $y = \log_2(8x)$, and $y = \log_2\left(\frac{x}{2}\right)$ are related to the graph of $y = \log_2 x$.

6. Evaluate the following logarithms.

a) $\log_{25} 5^3$

b) $\log_6 54 + \log_6 2 - \log_6 3$

c) $\log_6 6\sqrt{6}$

d) $\log_2 \sqrt{36} - \log_2 \sqrt{72}$

e) $\log_3 54 + \log_3\left(\frac{3}{2}\right)$

f) $\log_8 2 + 3\log_8 2 + \frac{1}{2}\log_8 16$

7. Use the laws of logarithms to express each of the following in terms of $\log_b x$, $\log_b y$, and $\log_b z$.

a) $\log_b xyz$

b) $\log_b\left(\frac{z}{xy}\right)$

c) $\log_b x^2y^3$

d) $\log_b\sqrt{x^5yz^3}$

8. Explain why $\log_5 3 + \log_5 \frac{1}{3} = 0$.

10. **A** Use the laws of logarithms to express each side of the equation as a single logarithm. Then compare both sides of the equation to solve.

a) $\log_2 x = 2\log_2 7 + \log_2 5$

b) $\log x = 2\log 4 + 3\log 3$

c) $\log_4 x + \log_4 12 = \log_4 48$

d) $\log_7 x = 2\log_7 25 - 3\log_7 5$

e) $\log_3 x = 2\log_3 10 - \log_3 25$

f) $\log_5 x - \log_5 8 = \log_5 6 + 3\log_5 2$

11. Write each expression as a single logarithm. Assume that all the variables represent positive numbers.

a) $\log_2 x + \log_2 y + \log_2 z$

b) $\log_5 u - \log_5 v + \log_5 w$

c) $\log_6 a - (\log_6 b + \log_6 c)$

d) $\log_2 x^2 - \log_2 xy + \log_2 y^2$

e) $1 + \log_3 x^2$

f) $3\log_4 x + 2\log_4 x - \log_4 y$

7.5 Solving Exponential Equations

#2, 3cdf, 4, 5, 6acd, 8adf, 10 (pg 485 in textbook)

2. Solve. Round your answers to three decimal places.

a) $2^x = 17$

b) $6^x = 231$

c) $30(5^x) = 150$

d) $210 = 40(1.5)^x$

e) $5^{1-x} = 10$

f) $6^{\frac{x}{3}} = 30$

3. Solve by rewriting in exponential form.

c) $x = \log_5 5\sqrt{5}$

d) $x = \log_2 \sqrt[5]{8}$

f) $x = \log_3\left(\frac{1}{\sqrt{3}}\right)$

4. The formula to calculate the mass, $M(t)$, remaining from an original sample of radioactive material with mass P, is determined using the formula $M(t) = P\left(\frac{1}{2}\right)^{\frac{t}{h}}$, where t is time and h is the half-life of the substance. The half-life of a radioactive substance is 8 h. How long will it take for a 300 g sample to decay to each mass?

a) 200 g

b) 100 g

c) 75 g

d) 20 g

5. **K** Solve.

a) $49^{x-1} = 7\sqrt{7}$

b) $2^{3x-4} = 0.25$

c) $\left(\frac{1}{4}\right)^{x+4} = \sqrt{8}$

d) $36^{2x+4} = \left(\sqrt{1296}\right)^x$

e) $2^{2x+2} + 7 = 71$

f) $9^{2x+1} = 81(27^x)$

6. A a) If \$500 is deposited into an account that pays 8%/a compounded annually, how long will it take for the deposit to double?

c) A \$5000 investment is made in a savings account that pays 10%/a compounded quarterly. How long will it take for the investment to grow to \$7500?

d) If you invested \$500 in an account that pays 12%/a compounded weekly, how long would it take for your deposit to triple?

8. Solve for x.

a) $4^{x+1} + 4^x = 160$ d) $10^{x+1} - 10^x = 9000$ f) $4^{x+3} - 4^x = 63$

10. Solve. Round your answers to three decimal places.

a) $5^{t-1} = 3.92$ c) $4^{2x} = 5^{2x-1}$

b) $x = \log_3 25$ d) $x = \log_2 53.2$

7.5 Solving Logarithmic Equations #1bef, 2bcf, 4ade, 5, 7, 9, 10, 11d, 12, 14 (pg 491)

1. Solve.

b) $\log_3 x = 4\log_3 3$ e) $\log_2 8 = x$ f) $\log_2 x = \frac{1}{2}\log_2 3$

2. Solve.

b) $\log_x 6 = -\frac{1}{2}$ c) $\log_5(2x-1) = 2$ f) $\log_5(2x-4) = \log_5 36$

4. Solve.

a) $\log_x 27 = \frac{3}{2}$ d) $\log x = 4$ e) $\log_{\frac{1}{3}} 27 = x$

5. K Solve.

a) $\log_2 x + \log_2 3 = 3$ d) $\log_4 x - \log_4 2 = 2$

b) $\log 3 + \log x = 1$ e) $3\log x - \log 3 = 2\log 3$

c) $\log_5 2x + \frac{1}{2}\log_5 9 = 2$ f) $\log_3 4x + \log_3 5 - \log_3 2 = 4$

7. Solve.

a) $\log_7(x+1) + \log_7(x-5) = 1$

b) $\log_3(x-2) + \log_3 x = 1$

c) $\log_6 x - \log_6(x-1) = 1$

d) $\log(2x+1) + \log(x-1) = \log 9$

e) $\log(x+2) + \log(x-1) = 1$

f) $3\log_2 x - \log_2 x = 8$

9. The loudness, L, of a sound in decibels (dB) can be calculated using
A the formula $L = 10 \log \left(\frac{I}{I_0}\right)$, where I is the intensity of the sound in watts per square metre (W/m^2) and $I_0 = 10^{-12}\ \text{W/m}^2$.

a) A teacher is speaking to a class. Determine the intensity of the teacher's voice if the sound level is 50 dB.

b) Determine the intensity of the music in the earpiece of an MP3 player if the sound level is 84 dB.

10. Solve $\log_a(x + 2) + \log_a(x - 1) = \log_a(8 - 2x)$.
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11. Use graphing technology to solve each equation to two decimal places.

d) $\log(4x) = \log(x + 1)$

12. Solve $\log_5(x - 1) + \log_5(x - 2) - \log_5(x + 6) = 0$.

14. a) Without solving the equation, state the restrictions on the variable x in the following: $\log(2x - 5) - \log(x - 3) = 5$

b) Why do these restrictions exist?